Signs of quadratic function

Part 6

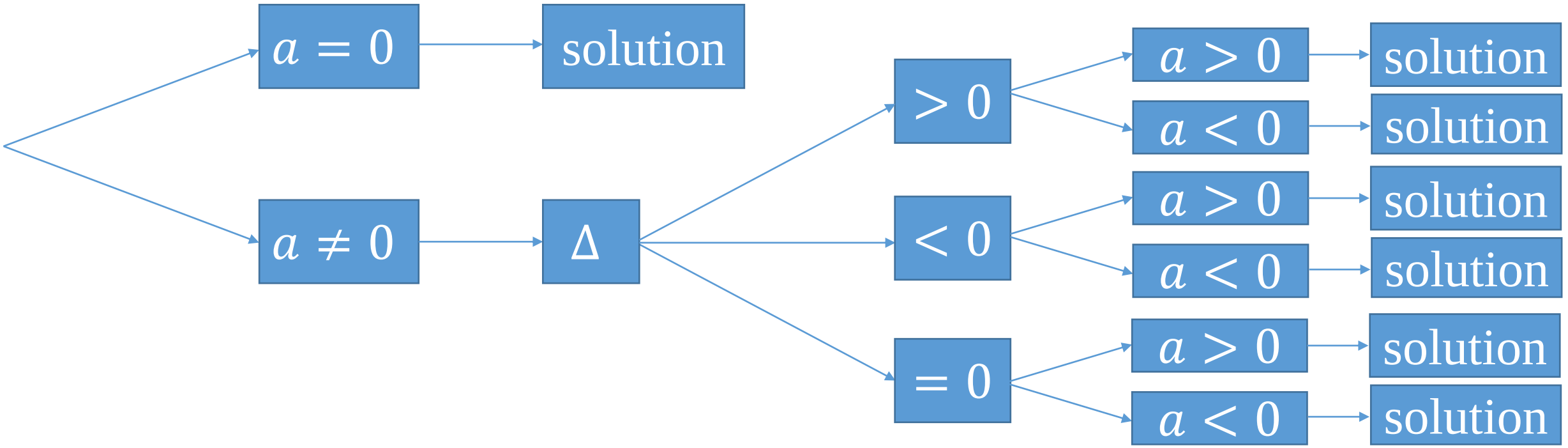
Parametric example

Consider the parametric inequality:

$$mx^2 - 2(m+1)x + m - 5 < 0 \quad ; m \in IR$$

The coefficients are: $a = m \; ; b = -2(m+1) \; ; c = m - 5$

Solving the parametric inequality needs the following:



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❖ $a = 0 \; ; \quad m = 0$

The inequality becomes: $-2x - 5 < 0 \quad ; \quad x > -\frac{5}{2}$

❖ $a \neq 0 \; ; \; m \neq 0$

$\Delta' = b'^2 - ac = (m+1)^2 - m(m-5) = 7m + 1$

m	$-\infty$		$-\frac{1}{7}$		0		$+\infty$
Δ'		$-$	0	$+$	‖	$+$	
a		$-$		$-$	‖	$+$	

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m	$-\infty$		$-\frac{1}{7}$		0		$+\infty$
Δ'		$-$	0	$+$	‖	$+$	
a		$-$	‖	$-$	‖	$+$	

$\Delta' > 0$: 2 distinct roots $x_1 = \frac{m+1-\sqrt{7m+1}}{m}$; $x_2 = \frac{m+1+\sqrt{7m+1}}{m}$

- $\Delta' > 0$ and $a > 0$: $m \in]0; +\infty[$

x	$-\infty$		x_1		x_2		$+\infty$
$f(x)$		$+$	0	$-$	0	$+$	

$S =]x_1; x_2[$

- $\Delta' > 0$ and $a < 0$: $m \in]-\frac{1}{7}; 0[$

x	$-\infty$		x_1		x_2		$+\infty$
$f(x)$		$-$	0	$+$	0	$-$	

$S =]-\infty; x_1[\cup]x_2; +\infty[$

Parametric example

m	$-\infty$		$-\frac{1}{7}$		0		$+\infty$
Δ'		$-$	0	$+$	‖	$+$	
a		$-$		$-$	‖	$+$	

$\Delta' < 0$: no real roots

- $\Delta' < 0$ and $a < 0$: $m \in]-\infty; -\frac{1}{7}[$

x	$-\infty$		$+\infty$
$f(x)$		$-$	

$$S = IR =]-\infty; +\infty[$$

Parametric example

m	$-\infty$		$-\frac{1}{7}$		0		$+\infty$
Δ'		$-$	0	$+$	‖	$+$	
a		$-$	‖	$-$	‖	$+$	

$\Delta' = 0$: one double root $x_1 = x_2 = -\frac{b'}{a} = \frac{m+1}{m}$

- $a = m = -\frac{1}{7} < 0$ $\quad x_1 = x_2 = -6$

x	$-\infty$		-6		$+\infty$
$f(x)$		$-$	0	$-$	

$$S = IR =]-\infty; -6[\cup]-6; +\infty[$$

Now it is your turn

Can you solve this parametric inequality????

$$mx^2 + 3x - m > 0 \quad ; \quad m \in IR$$